Q1

$\cancel{1} + \tan^2 2x = \cancel{1} + \tan 2x$ $\qquad \sec^2\theta \equiv 1 + \tan^2\theta$

$\tan^2 2x - \tan 2x = 0$

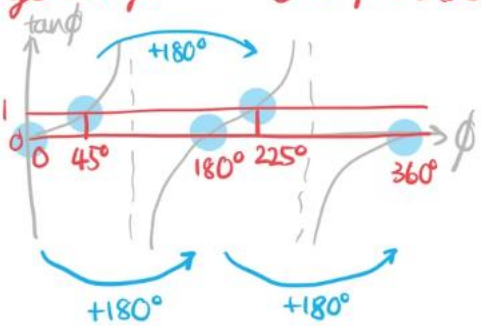
$\tan 2x(\tan 2x - 1) = 0$

$\tan(2x) = 0,\ 1$

$2x \to \phi$

$\phi = \tan^{-1}(0) = 0 \qquad \phi = \tan^{-1}(1) = 45$

Change range $\quad 0° \leq \phi \leq 360°$



$\frac{\phi}{2} = \boxed{x = 0°,\ 22.5°,\ 90°,\ 112.5°\ \ 180°}$

Q2

Given that

$$\sin(2A° - B°) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$2A° - B° \to \phi$

and that

$$3A = 4B \text{ and } 60° < B° < A° < 300°$$

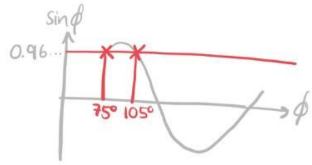
find the values of A and B.

$B = \frac{3}{4}A \qquad \phi = 2A - \frac{3}{4}A = \frac{5A}{4}$

$60 < A < 300$

$75 < \phi < 375$

$\sin^{-1}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) = 75$



$\phi = 75°, 105°$

$\frac{5A}{4} = 75° \qquad \frac{5A}{4} = 105$

$A = \cancel{60°} \qquad A = 84°$

reject since $A > 60°$ $\qquad B = \frac{3}{4}(84) = 63°$

Both A and B satisfy the inequality in the question.

$\therefore \boxed{A = 84°,\ B = 63°}$

Q3

Solve the equation

$$\frac{\cos x}{\operatorname{cosec} x} - \cot x = 0, \qquad -2\pi \leq x \leq 2\pi$$

$\operatorname{cosec} x \to \frac{1}{\sin x}$, $\cot x \to \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

[4]

$\cos^2\theta + \sin^2\theta \equiv 1$

$$\cos x \sin x - \frac{\cos x}{\sin x} = 0$$

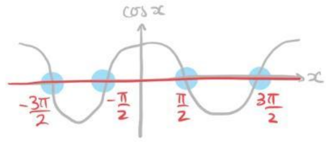
$$\cos x \sin^2 x - \cos x = 0$$

$$\cos x(1-\cos^2 x) - \cos x = 0$$

$$\cancel{\cos x} - \cos^3 x - \cancel{\cos x} = 0$$

$$\cos^3 x = 0$$

$$\cos x = 0$$



$$\boxed{x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}}$$

Q4a

a) $R\cos(A+B) = R\cos A\cos B - R\sin A\sin B$

$A \to \theta$, $B \to \alpha$

$\cos^2\theta + \sin^2\theta \equiv 1$

$$R\cos(\theta+\alpha) = 6\cos\theta - 8\sin\theta$$

$R\cos\alpha = 6 \qquad R\sin\alpha = 8$

Square and add

$$(R\cos\alpha)^2 + (R\sin\alpha)^2 = 6^2 + 8^2 = 100$$

$$R^2(\cos^2\alpha + \sin^2\alpha) = 100$$

$$R^2 = 100$$

$$R = 10$$

Divide

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{8}{6}$$

$$\tan\alpha = \frac{4}{3} \qquad \therefore \alpha = 0.927 \text{ (3sf)}$$

$$\therefore \boxed{6\cos\theta - 8\sin\theta = 10\cos(\theta + 0.927)}$$

Q4b

b) $5\cos(\theta + 0.927...) - 2 = 0$

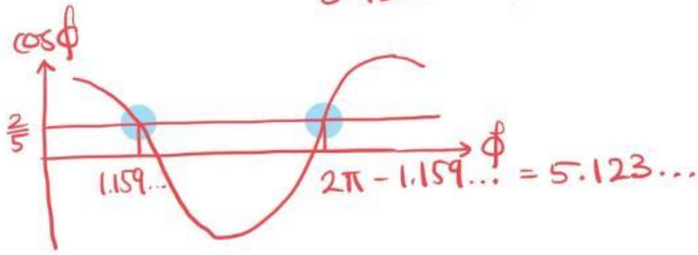
$\cos(\boxed{\theta + 0.927...}) = \frac{2}{5}$

let this angle = ϕ

$\phi = \cos^{-1}\left(\frac{2}{5}\right) = 1.159...$

change range: $0 + 0.927... \leqslant \phi \leqslant 2\pi + 0.927...$

$0.927... \leqslant \phi \leqslant 7.210...$



$\phi = 1.159..., \ 5.123...$

$\phi - 0.927... = \boxed{\theta = 0.232 \text{ and } 4.20}$ (3sf)

Q4c

c) Minimum value occurs when $\cos(\theta + 0.927...) = -1$

$\therefore$ $\boxed{\text{Min value: } -10}$

Smallest possible value of θ,

occurs when $\theta + 0.927 = \pi$

$\theta = \pi - 0.927$

$\boxed{\theta = 2.21}$ (3sf)

Q5

$\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$

$2(\operatorname{cosec}^2 x - 1) = 8 - \operatorname{cosec} x$

$2\operatorname{cosec}^2 x - 2 = 8 - \operatorname{cosec} x$

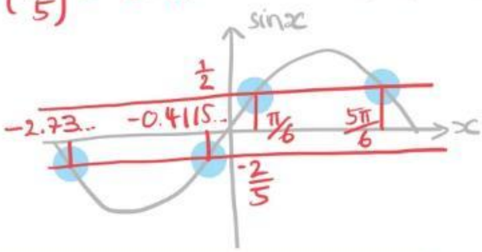
$2\operatorname{cosec}^2 x + \operatorname{cosec} x - 10 = 0$

$(2\operatorname{cosec} x + 5)(\operatorname{cosec} x - 2) = 0$

$\operatorname{cosec} x = -\frac{5}{2}\ ,\ 2$

$\sin x = -\frac{2}{5} \qquad \sin x = \frac{1}{2}$

$\sin^{-1}\left(-\frac{2}{5}\right) = -0.4115\ldots \qquad \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$



$\boxed{x = -2.73,\ -0.412,\ \frac{\pi}{6},\ \frac{5\pi}{6}}$

(3sf)

Q6

$8\cos^4\theta - 5(2\cos^2\theta - 1) - 2 = 0$ $\qquad \cos 2\theta = 2\cos^2\theta - 1$

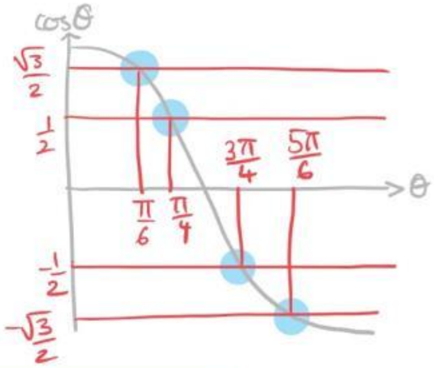
$8\cos^4\theta - 10\cos^2\theta + 3 = 0$

$(4\cos^2\theta - 3)(2\cos^2\theta - 1) = 0$

$\cos^2\theta = \frac{3}{4} \qquad \cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{\sqrt{3}}{2} \qquad \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \qquad \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$



$$\boxed{\theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{6}}$$

Q7

Determine the values of the constant k for which the equation

$$\operatorname{cosec}\theta = k, \qquad -\pi \leq \theta \leq 2\pi$$

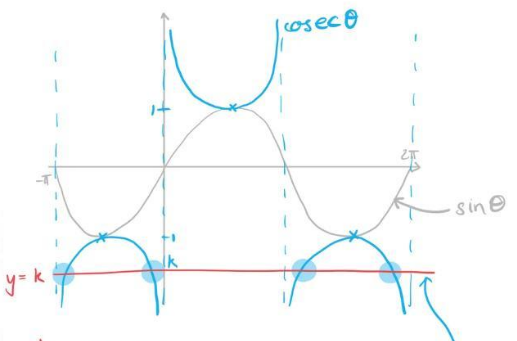
$= \frac{1}{\sin\theta}$

has (i) no real solutions,
(ii) 1 real solution,
(iii) 2 real solutions,
(iv) 4 real solutions

[4]

① Sketch $\sin\theta$, then use that to sketch $\operatorname{cosec}\theta$.

② Draw $y = k$ and shift the line up and down to find for which values of k the graphs intersect. The number of real solutions, is the number of points of intersection.



i) $-1 < k < 1$

ii) $k = 1$

iii) $k = -1$ or $k > 1$

iv) $k < -1$

4 points of intersection in the region $k < 1$
$\therefore$ 4 real roots.

Q8

$\operatorname{cosec}^2\theta - 1 = 15 - 6\operatorname{cosec}\theta$ $\qquad \operatorname{cosec}^2\theta \equiv \cot^2\theta + 1$

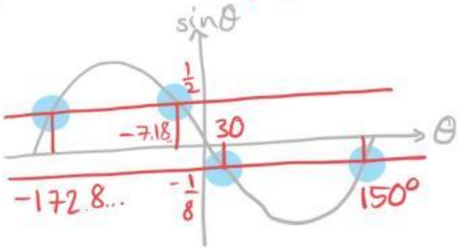
$\operatorname{cosec}^2\theta + 6\operatorname{cosec}\theta - 16 = 0$

$(\operatorname{cosec}\theta + 8)(\operatorname{cosec}\theta - 2) = 0$

$\operatorname{cosec}\theta = -8 \qquad \operatorname{cosec}\theta = 2$

$\sin\theta = -\frac{1}{8} \qquad \sin\theta = \frac{1}{2}$

$\sin^{-1}\left(-\frac{1}{8}\right) = -7.18\ldots \qquad \sin^{-1}\left(\frac{1}{2}\right) = 30$



$$\boxed{\theta = -172.8^\circ, -7.2^\circ, 30^\circ, 150^\circ}$$